THERMO-MECHANICAL THEORY OF NON-NEWTONIAN FLUIDS

Final Report for the period July 1, 1964 - June 30, 1965

Grant Number: NsG 664

Contractor: Princeton University

Address: Princeton, New Jersey

Investigators: Principal: D. C. Leigh[1]
Associate: J. C. Wang

Objectives:

(1) Formulation of thermodynamic theory for general non-Newtonian fluids.

(2) Investigation of the consequences of thermodynamic principles and theory on constitutive equations and on the solution of specific problems.

(3) Study of flows of non-Newtonian fluids in which thermal variables and their distributions are important.

Background:

Thus far in the theoretical developments of constitutive equations and solutions for non-Newtonian fluids, thermal effects have been largely ignored and omitted. This has not been because of a lack of appreciation of thermal effects, but rather because the development of purely mechanical theories of nonlinear fluids was difficult enough and also because one was not quite sure what the governing thermodynamic principles were for non-linear materials.

Recently an important contribution to objective (1) above has been

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made by Coleman[2] who went a long way towards formulating a thermodynamic theory of general nonlinear materials with memory. That theory attempts to provide a thermodynamic framework within which specific constitutive equations may be studied.

Work on this grant can be divided into two projects:

Project (i) The study of the stability and sign of the rate-of-deformation work for the solution of boundary value problems of second-and-third order Coleman-Noll fluids.

Project (ii) The study of the thermodynamics of materials with memory the development of an approximation theory including thermodynamics, and application to non-Newtonian fluids.

Results: The results to-date fall under objectives (1) and (2) above.

Project (i) This project was largely carried out during the first six months of this grant and is summarized in Ref. 1. Detailed reporting is contained in Refs. 2 and 3.

Four problems in unsteady incompressible simple-shear flow were considered. Solutions ignoring thermodynamics, for the second order fluid approximations were listed, including one which appeared to be new. For the third-order fluid approximation the governing differential equation is nonlinear; solutions were obtained to appropriate linearized problems. The solutions were examined with regard to stability and how it relates to the approximation theory. The sign of the rate of deformation work was

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[2] B. D. Coleman, Arch. Ratl. Mech. Anal., 17, 1 (1964).

investigated not only as it depends on the material constants but also as it depends on the problem solution. Implications for a complete thermodynamic solution are discussed.

Project (ii) This project was largely carried out during the second six months of this grant. A preliminary draft of a paper has been written and is currently undergoing revision. A summary of the results of this investigation follows:

The thermodynamic theory of materials with memory is developed in terms of past histories. A modification of Coleman's postulate of the Second Law of Thermodynamics is put forth. The Coleman-Noll approximation theory for materials with memory is developed for the thermodynamic theory. The foregoing results are specialized to fluids. For linear viscous fluids the classical results are obtained, except for the constitutive equation for the entropy. The entropy constitutive equation contains, in addition to the classical perfect fluid term, a term linear in the time derivative of the temperature. For the incompressible second-order fluid many restrictions on and relations between the material coefficients are obtained. In particular, we find that the entropy production for second-order fluids is the same as that for linear fluids and therefore that all smooth admissible motions of second-order fluids are allowed.

References:

1. D. C. Leigh, Progress Report for the period July 1, 1964 - December 31, 1964, NASA Grant NsG 664 on "Thermo-Mechanical Theory of non-Newtonian fluids" with Princeton University.

2. D. C. Leigh, "Thermodynamics and Higher-Order Fluid Theories", NASA CR - 292, August, 1965.

3. D. C. Leigh, "Stability and Dissipation in Second and Third-Order Fluid Approximations," accepted for presentation at the Annual A.J.Ch.E Meeting Philadelphia, December 6-10, 1965.

October 12, 1965

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